A. ANTONIOUK [1]

Least Action Principle for Weighted Porous Media Equation

[1] *Institute of Mathematics NAS Ukraine, Kyiv, Ukraine*
E-mail: antoniouk.a@gmail.com

In the beginning 18th century Leibniz, Maupertuis, Euler claimed that all physical phenomenons may be obtained from the Least Action Principle and since Lagrange and Hamilton it was well understood for the classical mechanics. However only in 1966 V.I. Arnold in [2] achieved it for the fluid dynamics. To do this he remarked that the group of volume preserving diffeomorphisms $\mathcal{D}_\mu(M)$ of a manifold M (μ being a given volume element on M) is the appropriate configuration space for the hydrodynamics of an incompressible fluid. In this framework the solutions to the Euler equation become geodesic curves with respect to the right invariant metric on $\mathcal{D}_\mu$.

The main our result [1] shows that the weighted porous media equation ([3], [4]), which generalizes the standard porous media equation,

$$\frac{\partial u}{\partial t} = \left(-u \cdot \nabla + \frac{1}{2}\Delta\right)\left(\|u\|^{q-2}u\right) + \nabla P$$

may be also obtained in the framework of Least Action Principle for specially chosen energy functional. In the particular case of $q = 2$ this recovers the Navier-Stokes equation.

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