Some Properties of Orthogonal Wavelets Based on Jacobi Polynomials

Vasyl Semenov

Scientific-Production Enterprise Delta SPE, KIev, Ukraine,
Institute of Hydromechanics of NASU, Kiev, Ukraine, mlev@bk.ru

In this study we investigate wavelets based on Jacobi polynomials. We use the definition of n-th order wavelet as in [1]:

$$\psi_{n,r}(x) = \sum_{k=n+1}^{2n} P_k(t_r) P_k(x) \tag{1}$$

for some fixed set of parameters $t_1, t_2, \ldots, t_n$. The definition of n-th order Jacobi polynomial depending on the parameters α, β is as follows

$$P_n^{(\alpha,\beta)}(x) = \frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \sum_{m=0}^{n} \binom{n}{m} \frac{\Gamma(\alpha+\beta+n+m+1)}{\Gamma(\alpha+m+1)} \left(\frac{x-1}{2}\right)^m,$$

where $\Gamma()$ is a gamma function.

In this investigation we consider the following questions regarding wavelets (1):

1. **Linear dependence.** For which t_r, $r = 1, ..., n$, the wavelets $\psi_{n,r}(x)$ are linearly dependent?
2. **Orthogonality.** For which t_r, $r = 1, ..., n$, $\psi_{n,r}(x)$ are orthogonal?
3. **Riesz stability.** For which t_r, $r = 1, ..., n$, the minimum Riesz stability constant is achieved?

This investigation is a joint work with Professor Juergen Prestin (Universität zu Lübeck, Institut für Mathematik, Lübeck, Germany).

[1] B. Fischer, J. Prestin, Wavelets based on orthogonal polynomials, *Mathematics of computation*, **66** (1997), p. 1593–1618.