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# Mathematical model of immune response with the influence of external factors

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G.I. Marchuk proposed his mathematical model of immune reaction [1] in the eighties of past century. The simplest version of the model describes humoral response, with the assumption of constant environmental conditions implying constant parameters of the model. The first attempt to model the influence of external conditions could be found in [2], where periodic seasons are reflected by periodic model coefficients. We investigate the influence of environmental pollution on the course of immune reaction described by the Marchuk model, that is we consider mathematical model that reads

$$\begin{aligned} \dot{V} &= \beta(1-\delta V^n)V - \gamma V F, \\ \dot{C} &= \alpha\xi(m)V_\tau F_\tau - \mu(C-C^*) - \mu_1(E-1), \\ \dot{F} &= \varrho C - \eta\gamma F V - \mu_f F, \\ \dot{m} &= \sigma V - \mu_m m + \mu_2(E-1), \\ \dot{E} &= r(1-E_\Delta)E, \end{aligned} \tag{1}$$

where $V(t)$, $C(t)$ and $F(t)$ reflect the amount of antigen, plasma cells and antibodies, respectively, $m(t)$ is the characteristic of organ-target, $\tau$ is the delay of immune response reflecting the time needed to form the cascade of plasma cells, $E(t)$ is the average indicator of pollution with the delay $\Delta$ characterising the average recovery time of ecological balance.

For the model described by Eqs. (1), stationary solutions were found, their stability was investigated and numeric modelling was held.

[1] G. I. Marchuk, *Mathematical Modelling of Immune Response in Infectious Diseases*, Springer-Verlag, New York, 1997.

[2] M. Bodnar, U. Foryś, A model of immune system with time-dependent immune reactivity, *Nonlinear Analysis-Theory Methods & Applications* **7(2)**, (2009), pp. 1049-1058.