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Some extremal problems in Orlicz spaces

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Let $(\mathbb{R}^m, d\mu)$, $m \geq 1$, be the m-dimensional Euclidean space of points $\mathbf{x} = (x_1, \ldots, x_m)$ equipped with finite σ-additive continuous measure $d\mu$, let $\mathbb{A}$ be a μ-measurable subset of $(\mathbb{R}^m, d\mu)$ whose μ-measure is equal to $a \in (0, +\infty]$. An Orlicz function $M(t)$, $t \geq 0$, is a non-decreasing convex down function satisfying the conditions $M(0) = 0$ and $M(t) \to +\infty$ as $t \to +\infty$. Denote by $L_M := L_M(\mathbb{A}; d\mu)$ the set of all functions f defined on $\mathbb{A}$, measurable with respect to the measure $d\mu$, such that $(\forall C{>}0) : \int_{\mathbb{A}} M(C|f(\mathbf{x})|)\, d\mu < +\infty$. Equipped with the norm $\|f\|_{L_M} := \inf\{\alpha{>}0 : \int_{\mathbb{A}} M(|f(\mathbf{x})|/\alpha)\, d\mu \leq 1\}$ the linear space L_M is the Banach space and called Orlicz space. Also we set $L_p := L_M$, when $M(t) = t^p$, $0 < p < \infty$, and denote by $U_p^+(\mathbb{A})$ the subset of all nonnegative functions from unit ball of L_p, $\mathcal{U}_p := U_p^+ \cap L_M$.

Further, let $f \in L_M$ and $\sigma > 0$. Consider the quantities

$$e_\sigma(f)_{L_M} := \inf\{\|f - \chi_{\gamma_\sigma} f\|_{L_M} : \mathrm{mes}_\mu \gamma_\sigma = \sigma\},$$

where $\chi_{\gamma_\sigma} = \chi_{\gamma_\sigma}(\mathbf{x})$ is the characteristic function of the set γ_σ ($\chi_{\gamma_\sigma}(\mathbf{x}){=}1$, when $\mathbf{x} \in \gamma_\sigma$ and $\chi_{\gamma_\sigma}(\mathbf{x}) = 0$, when $\mathbf{x} \notin \gamma_\sigma$).

Theorem 1 *Suppose that $p \in (0, \infty)$, φ is a nonnegative essentially bounded on $\mathbb{A}$ function that $\lim_{|\mathbf{x}| \to +\infty} \varphi(\mathbf{x}) = 0$, when $\mathrm{mes}_\mu \mathbb{A} = +\infty$, and M is Orlicz function such that the function $M(t^{1/p})$ also is Orlicz function. Then for any $\sigma \in (0, a)$,*

$$e_\sigma(\varphi, p)_{L_M} := \sup_{y \in \mathcal{U}_p} e_\sigma(\varphi\, y)_{L_M} = \sup_{s \in (\sigma, a]} \Big(\int_0^s \bar{\varphi}^{-p}(t) dt \Big)^{-\frac{1}{p}} \Big(M^{-1}\big(\frac{1}{s - \sigma}\big) \Big)^{-1},$$

where M^{-1} is the converse function of the function M, $\bar{\varphi}(t)$ is the decreasing rearrangement of $\varphi(\mathbf{x})$. The least upper bound on the right-hand side of the last relation is realized for a certain finite value $s = s^*$.

Note that in the case, where $M(t) = t^q$, for all $0 < q < \infty$, similar assertions were obtained by A.I. Stepanets and A.L. Shidlich.