

On spaces extremal for the Gomory-Hu inequality

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Let (X, d) be a finite nonempty ultrametric space. In 1961 E. C. Gomory and T. C. Hu [1] proved the inequality $|\text{Sp}(X)| \leq |X|$ where $\text{Sp}(X) = \{d(x, y) : x, y \in X\}$. Using weighted Hamiltonian cycles and weighted Hamiltonian paths we give new necessary and sufficient conditions under which the Gomory-Hu inequality becomes an equality. We find the number of non-isometric (X, d) satisfying the equality $|\text{Sp}(X)| = |X|$ for given $\text{Sp}(X)$. Moreover it is shown that every finite semimetric space Z is an image under a composition of mappings $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ such that X and Y are finite ultrametric space, X satisfies the above equality, f is an ε -isometry with an arbitrary $\varepsilon > 0$, and g is a ball-preserving mapping.

- [1] R. E. Gomory and T. C. Hu., *Multi-terminal network flows*, SIAM, **9**(4), (1961), pp. 551–570.
- [2] O. Dovgoshey, E. Petrov, and H. M. Teichert, *On spaces extremal for the Gomory-Hu inequality*, P-Adic Numbers, Ultrametric Analysis, and Applications, **7**(2), (2015), pp. 133-142.